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On the Conceptual Nature of the Physical Constants*

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Introduction.

In most formulae of physics, or, more generally, in most theoretical analyses of any physical phenomenon, there appears one or more physical constants. Some of these play an essential and pervasive role in physics. They are variously called "general", or "fundamental", or "universal" physical constants. Yet, despite their importance, very little seems to have been written about their nature and significance until rather recently [1] [2].

A quick glance at the contents of different tables of such constants, however, should be sufficient to raise up several questions, bearing upon some deep conceptual aspects of physics. Consider, for instance, the seminal article written in 1929 by Birge [3], one of the first specialists in the systematic investigation of the physical constants. In this paper, which rightly starts by asserting that "some of the most important results of physical science are embodied, directly or indirectly, in the numerical magnitude of various universal constants", the determination is studied of the following constants: velocity of light c, gravitation constant G, relation of litre to cubic centimetre, normal mole volume of ideal gas V_0, relation of international to absolute electrical units, several atomic weights (H, He, N, Ag, I, C, Ca), normal atmosphere, absolute temperature of ice-point, mechanical equivalent of heat J, Faraday constant F, electronic charge e, specific charge of the electron e/m, Planck constant h plus various "additional quantities" (ratio of e.s. to e.m. units, density of water, Rydberg

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constant, Avogadro number N, Boltzmann constant k, etc.). Thirty years later, Cohen, Crowe and DuMond [4] in their book, *The Fundamental Constants of Physics,* distinguish "classical constants" such as G, V_0, R, J, F from "atomic constants", while recognizing that "the meaning of the term 'atomic constants' has become increasingly inclusive and indefinite". In 1969, Taylor, Parker and Langenberg [5] chose as "*the* fundamental physical constants" the following set: c, e, h, N, a.m.u., m_e, M_p, M_n, k, G; see also the table compiled by Cohen in 1974 [6]. More recently, Mohr and Taylor [7] restrain the list of "universal constants" to c, G, h and (curiously) ε_0 and μ_0, while as of 2015, the Wikipedia article sticks to c, G, h. The heterogeneity and variability of these lists offers a relevant starting point for our reflections.

Here are some of the questions I will attempt to answer in this paper:

— why are there "fundamental constants" in physics and not, for instance, in biology or geology?

— why are there no such constants in the most classical theories of physics, such as classical mechanics?

— are not the classical constants of thermodynamics and statistical mechanics, R (or k) and J, less fundamental than the "atomic" modern constants, c and h?

— is there anything common between a simple unit conversion factor, such as the ratio of litre to cubic centimetre, and a universal constant, such as Planck's?

— why is the velocity of light c considered as a fundamental constant when, according to its very name, it seems to be associated with a particular class of physical phenomena only, namely the propagation of electromagnetic radiation?

— what is the meaning of taking the value of some of these constants as unity, as though this value was not to be experimentally determined?

— conversely, how can one let these constants "go to zero" (or to infinity), as if they were not *constants,* in order to define approximate limiting theories such as Newtonian mechanics ($h \to 0$) and Galilean relativity ($c \to \infty$)?

— are all the so-called fundamental constants on the same footing, whether they be masses of elementary particles, coupling constants or unit conversion factors, such as M_p, G and the a.m.u. (atomic mass unit) respectively, to stay within the list of "atomic" constants?

— how comes that the contents of the tables of "fundamental" physics constants vary with time, as a simple comparison of various such tables reveals (see above)?

I will try to show that the answers to these questions and other ones rely on the understanding of physical science as a historical process. Only by studying the conditions for the appearance, or disappearance, of physical constants can we understand their nature. Only by emphasizing the variations in status of a given constant, can we understand its role. Only by contrasting the opposite effects of theoretical and experimental practices upon the fate of such a constant, can we analyse its significance. The present investigation thus takes place within a definite vision of physics, and science in general, as a social endeavour. Its ensuing historicity should then be put into light even at its seemingly most abstract and formal levels. The case of physical constants thus epitomizes this view, since their constant numerical values make sense only through a changing conceptual nature.

I. – A classification of physical constants.

Let me start by proposing a classification of physical constants into three types. This, hopefully, may bring some order in the otherwise rather incongruous lists offered by the standard tables. By order of increasing generality,I will thus distinguish:

A) *Physical properties of particular objects*: for instance, the masses of fundamental particles, their magnetic moments, energy widths of unstable ones, etc.

B) *Constants characterizing whole classes of physical phenomena*: these are mainly the coupling constants of the various fundamental interactions, such as Newton's constant G associated with gravitation.

C) *Universal constants,* such as *c* or *h,* which enter the most general theoretical framework available, independently of particular objects or specific interactions (I will come back later on to the validity of approximate theories, where such constants may be neglected — see section 3).

The interest of such a classification is not to offer an intrinsic, absolute and invariant characterization of any given constant. Quite on the contrary, it is its strong time dependence which makes it useful for discussing the changing status of most physical constants. Indeed, mobility is the rule, a constant moving from one type to another when our physical knowledge increases. Consider first the constants of type-A. While for quite a few decades the masses of the nucleons (for instance) belonged to that class, we are now convinced that their values can (or could) be explained in terms of the masses of their constituents (quarks, gluons) and the strengths of their interactions. The nucleon masses thus in some sense may be dropped out altogether from the table of fundamental constants, their status now being that of derived quantities. This is the case even though one is not yet able to compute them exactly from the deeper theory: the principle of their dependence upon more fundamental quantities is sufficient to ensure their ”de-fundamentalisation”. This is precisely what has happened before to the old physical constants, which, at the beginning of this century, consisted of the macroscopic properties of the simple elements, such as density, compressibility or heat capacity. Now we know that their values rely on the atomic structure of matter and are explainable in principle from quantum theory, even though very few such calculations may be achieved in fact. The same happened for the atomic and molecular properties, such as ionization energies or polarizability, and then for nuclear properties, such as masses, sizes, etc. The new fundamental constants, in terms of which the old ones are explained away, may belong to any of the three classes. The atomic and molecular quantities thus are eliminated once they are known to depend on the electronic mass m (type-A), on the electromagnetic coupling constant e (type-B) and on Planck quantum constant (type-C). In the same way, the advent of quantum electrodynamics has enabled us to express the electron magnetic moment in terms of the same quantities, so that it is no longer a fundamental constant.

But a type-A constant, instead of vanishing from the table, may be promoted to another category. This is the case of e, for instance. First characterized as the electric charge of the electron, a specific property of a particular object, it was later on recognized as the coupling constant of electromagnetic fields to all charged fundamental constituents of matter, and associated with the whole class of all electrodynamic phenomena. It thus became a type-B constant. An even more important example is afforded by the change in status experienced by c. As the terminology unfortunately still reflects, c was first introduced as the speed of light, that is a type-A constant. With the development of electrodynamics (classical), it came to be understood as playing a role in all electromagnetic phenomena: in most theoretical expressions, its significance is not directly that of a velocity (even though its dimensions are, of course), and one might thus think of c as a type-B constant. But the advent of Einsteinian relativity forces us to associate c with the theoretical description of space-time itself, independent of its specific contents. This is proved by the fact that Einsteinian relativity, according to our present knowledge, rules all fundamental interactions, implying the occurrence of c in the relevant theories even when no electromagnetic phenomena are to be considered at all. This point is blurred by the traditional terminology ('speed of light'), associated with a operational interpretation of relativity theory, whereby the Lorentz transformations are derived from an analysis of communication through electromagnetic signals. The theory, however, may be built upon a structural analysis of space-time, without using any postulate about the velocity of light [8]. That c thus has to be considered as a type-C constant, and not a type-A only, may be further emphasized by stressing that it could well be the velocity of …no existing physical object. If the photon had a non-vanishing mass, however small, its velocity would be closely approximated by c in all presently known situations, but would differ from it for low enough energies [9]. While such an occurrence would not *per se* ruin the validity of Einsteinian relativity, it would, however, invalidate most of its customary derivations. As a last argument, one might think of how Planck constant would have been considered, had it been first introduced through the discovery of angular-momentum quantization for the photon; it would then probably go by the name of the "spin of light". In fact, c is not the speed of light any more than h is the spin of light. Renaming it "Einstein constant" would certainly be appropriate

The same phenomena of elimination or promotion may affect type-B constants. Indeed, if a theoretical unification of two classes of interactions is realized, one (or perhaps both) of the coupling constants will lose its (their) fundamental nature, in favour of the other one (or of another, new, constant). Such a phenomenon was witnessed in the past with the unification by Maxwell of electricity and magnetism, whereby the magnetic permeability μ_0 and the electric permittivity ε_0 of the vacuum (type-B constants) were found to be related through the "speed of light" c (type-A constant). The same situation in some sense is realized by the unification in the Standard Model of the weak and electromagnetic constants. In the case in which all four (or more) fundamental interactions would be unified, they would be described by a new fundamental universal constant, of type-A.

A similar case would be realized if some of the constants were shown *not* to be constants, exhibiting a cosmological time dependence as in the now abandoned Dirac hypothesis [10]. The new constant parameters in terms of which the time dependence of these no longer constants would be expressed, should then take their place in the tables.

Not only does the type of a fundamental constant (or its absence thereof) depend on the history of physics, but it may also vary according to one's implicit epistemological position. This remark is already clear from our discussion of *c*. But a better case in point, since a more controversial one, is that of Newton's gravitational constant *G*. According to the standard point of view upon general relativity, space-time itself is ruled by gravity along with all phenomena within it. General relativity, in its geometrical interpretation, is an all-embracing theory, and its characteristic constant *G* should thus be elevated to type-C dignity. However, there exists an heterodox point of view, according to which the so-called "general relativity" is but a particular theory of a spin-2 classical field [11]. This field is universally coupled to energy, including its own, which endows it with a specific nonlinear behaviour. Because of this universal coupling, furthermore, the field plays the role of an effective variable space-time Riemannian metric ruling all physical phenomena. Needless to say, the formal theory is exactly identical to the conventional one, so that no experimental discrimination is possible. The advantage of such a view is to maintain gravitation at the level of the other fundamental interactions, its theoretical description being given by a local field theory as well. The price to be paid is the loss of the *a priori* intrinsic geometrical interpretation. Conversely, this unconventional point of view offers more room for modifying the theory if experimental results some day require such a change. In any case, within this framework, *G* keeps its pre-Einsteinian type-B status, on the same footing as other coupling constants.

Let us, from now on, concentrate on the universal physical constants (type-C).

2. – **The fate of universal constants.**

2·1. *Conceptual synthesis and analysis.* In order to understand the role of the universal physical constants, let us consider the particular case of Planck constant h. It was first introduced into physics through the Planck-Einstein relationship, $E = h\nu$[‡]. This relationship is customarily interpreted as associating an energy E with the frequency ν of an undulatory physical phenomenon. The connection thus established between a concept of particle mechanics, the energy of a discrete entity and one of wave theory, the frequency, leads to the idea of wave-particle duality in quantum physics, and, further on, to the philosophy of complementarity. Such an interpretation was quite natural in the early days of quantum theory, when one had to approach this new unknown theory from the old classical ones. Duality and complementarity served the very useful purpose of letting physicists use the classical concepts in the quantum domain as far as possible, while taking into account their limits of validity as imposed by these general principles. In such a way, many quantum results were obtained, or at least qualitatively understood, without using a yet-to-be-developed full quantum theory. Most of Bohr's theoretical work is a magnificent example of such a line of thought. It is to be realized today, however, that quantum theory does exist and that its concepts, after a century of collective practice, are deeply rooted in the present common sense of working physicists. These concepts need no longer be approached from classical ones, but may, and should, be taken at their face value. Such an intrinsically quantum understanding leads one to recognize that the objects of quantum physics are not either waves or particles, as duality would want us to believe; they are *neither waves, nor particles,* even though they do exhibit, under very particular circumstances, two types of limit behaviour as (classical) waves, or (classical) particles (see sect. 3.) It has been proposed to stress this ontological point by calling them "quantons" [13, 14]. Coming back to Planck's constant, the relationship $E = h\nu$, according to this point of view, is not to be interpreted as linking two classical concepts, but rather as transcending them through their synthesis, to establish a new single concept with a broader scope. The quantum energy indeed is a new concept, since it associates to

‡ We will refrain here to discuss why the reduced Planck constant defined by $\hbar = h/2\pi$ yields a better "quantum constant" than the original Planck constant h (see [12]).

any physical state a whole spectrum of numerical values and has to be represented by a Hermitian operator, as opposed to the single-number function which represents energy in classical mechanics. Here again a new name should have been given to stress the emergence of this concept, as an intrinsic one. Energy and frequency then appear as two particular facets of a more general notion, each of which being the only visible one from either one of two quite specific points of view.

The role thus played by Planck constant in bringing together these two facets is characteristic of universal constants. Any universal constant may be so described as a "concept synthesizer", expressing the unification of two previously unconnected physical concepts into a single one of extended validity. It will be shown below that classical constants such as k and J play exactly the same role. In any case, the same analysis may be applied to c, one use of which, for instance, is to bring together the concepts of spatial intervals Δx, on the one hand, and time intervals Δt, on the other hand. These are but two aspects of the more general notion of a space-time interval $\Delta s = (\Delta t^2 - c^{-2} \Delta x^2)^{1/2}$, which reduces to one or the other under special circumstances. Any universal constant usually brings about several such synthetical concepts. Planck constant also unifies momentum and wavenumber through the de Broglie relationship $p = h/\lambda$, while c unifies mass and energy through the Einstein relationship $E = c^2 m$. This is easily understood, since any physical concept by essence belongs to a theoretical framework which relates it to other concepts. The synthesis of two concepts thus is a local aspect of a more global unification of two pre-existing consistent theoretical structures. Bringing them into contact at one point usually requires the fitting together of other parts as well. This is what happens when the spatio-temporal consistency of particle mechanics, on the one hand, and wave theory, on the other, requires h to play the same role with respect to momentum and wave number (space aspect) as it does with respect to energy and frequency (time aspect). Fully endorsing this point of view may lead to a better understanding of the new concepts. For instance, it enables one to stress that the de Broglie relationship, $p = h/\lambda$, exhibits the intrinsically nonclassical nature of the quantum concept it establishes. Indeed, the classical wavelength is independent of the reference frame, or Galilean invariant, while the momentum of a classical particle with mass m changes according to

$p' = p + mv$, under a change of Galilean frame with velocity v. The de Broglie relationship thus seems inconsistent with Galilean invariance, that is, with the structure of space-time in "nonrelativistic" physics [15]. This pseudoparadox, far from dis- missing the de Broglie relationship as Lande maintained, points to a conceptual difference between classical wave number which indeed is Galilean invariant, and quantum "wave number" which is not [16]. The quantum "wave-length" transforms according to $1/\lambda' = 1/\lambda + mv/h$, which does *not* reduce to the classical limit $\lambda' = \lambda$, when h "goes to zero" (see below, section 3). This fact is related to the quantum "waves" being represented by complex numbers, in contradistinction to the real amplitudes of classical waves (whether they be acoustical, or hydrodynamical). As another example of the accent put on the conceptual nature of h as a building tool of quantum theory, one may write a third relationship on the same footing as the Planck and de Broglie relationships, but concerning now the angular momentum which leads to a heuristic understanding of the discretization of the quantum angular momentum [17].

Universal constants thus express synthetical transcending not of isolated pairs of concepts, but of whole conceptual arrays. In this sense, a universal constant is a "theory synthesizer", more than a mere "concept synthesize". From this abstract point of view, the various specific syntheses expressed through a universal constant between various pairs of concepts belonging to two theoretical frameworks are but equivalent consequences of the general theoretical unification of these frameworks. Nevertheless, because of historical considerations and epistemological motivations, they are not actually given an equal status, especially in educational practice. Some of them are taken as a starting point or fundamental hypothesis, such as $E = h\nu$, or $\Delta s = (\Delta t^2 - c^{-2} \Delta x^2)^{1/2}$, while other ones are considered as derived relations, or consequences, such as $p = h/\lambda$, or $E = c^2 m$. Since one has to start somewhere, it is probably true that the equivalence of all such expressions, as reflecting various aspects of one and the same synthetical process through a given universal constant, is bound to remain a rather abstract statement. Its acceptance, however, may pave the way to a modification of the traditional hierarchy. As an example, it has been proposed to develop Einsteinian relativity by starting directly from the mass-energy relationship $E = c^2 m$, by building upon it the "relativistic" concepts of energy and momentum, and then by deriving from them the theoretical structure of space-time [18]. After all, this corresponds more closely to the real needs of physics where the Lorentz-transformation formulae or invariant expressions are actually more often used for momentum-energy quantities than for space-time ones. Moreover the role of c in relating energy and inertia (rather than mass) is deeply rooted in the immediate prehistory of Einsteinian relativity, and "could" have been the starting point of an alternative historical path towards this theory. These considerations, clearly, are of some epistemological and pedagogical importance [19].

We are now in a position to answer the question of the existence of universal constants in physics as distinct from other sciences. This is simply due to the fact that in physics alone do the scientific concepts show an intrinsically mathematical expression. In physics, mathematics does not simply apply; it plays a far deeper, constitutive role [20]. The identification, or synthesis of two concepts in physics thus requires first their mathematical nature to be identical (scalars or vectors, for instance) and then implies the existence of a proportionality factor. Let me stress that numerical measurements of a given quantity, as may exist in other sciences (even social ones), are not sufficient to endow it with mathematical constitutivity; it is necessary that there exist nontrivial mathematical relationships between several such quantities, expressing the "scientific laws" of the field.

But the role of universal constants in the synthesis and unification of previously unrelated concepts or sets thereof, if it is the prime one in historical order of appearance, has for a corollary the fact of their leading to split and separate previously fused, if not confused, concepts. Two simple examples in relativity theory may be given here. The first one deals with the impossibility in Einsteinian relativity of a concept with the following two properties possessed by the velocity in Galilean relativity: i) being an additive quantity, obeying the simple composition law $v_{12} = v_1 + v_2$, and ii) giving the time rate of spatial change, namely $v = dx/dt$, for uniform motion. In Einsteinian relativity, if the second property is used as a definition of what we will keep calling "velocity" v, the first one will hold true for another quantity, the so-called "rapidity" φ. The two quantities are related by $v = \tanh\varphi$, or, with dimensional notations, by $v = c\tanh(\varphi/c)$, which makes apparent their fusion in the limit $c \to \infty$. The introduction of the concept of rapidity is of a major help for educational purposes [21]. It not only yields a more compact and more significant expression for Lorentz transformations via hyperbolic functions, but it explains away the pseudoparadoxes associated to the idea of a limiting velocity or nonadditivity of velocities, as simply due to a bad choice of parameter, such as would occur if rotations were labelled through the tangent of the angle instead of the angle itself. In recent decades, the concept of rapidity has also been fruitfully used in high-energy phenomenology[§]. A similar clarification may be achieved in relativistic dynamics, by introducing, with the concepts of energy and mass, the one of inertia, defined as the coefficient of the velocity in the expression for the moment. It is seen then that inertia is to be identified with energy in Einsteinian relativity, but with mass in Galilean relativity. The occurrence of the universal constant c thus splits inertia from mass as it fuses it with energy. In the description of space-time, the splitting of the categories of simultaneous pairs of events from that of null (lightlike) intervals may be interpreted in quite the same way. Other examples can be found at will. To use the same material metaphor as above, it may be said that the fitting of two conceptual structures, while bringing into contact previously separated pieces, also generates stresses requiring various splits within the new body.

2·2. *Units and unity.* It is an elementary, but crucial remark that the role of a universal constant as underlying the foundations of new concepts systematically decreases in importance when time goes on and the novelty of the concepts fades away. Indeed, when a sufficient familiarity has been acquired through years of experimenting, theorizing and teaching, one no longer needs to reach these concepts through the relationship of the ancient ones as synthesized by the universal constant. One simply uses the concepts as such. The constant then appears as a mere numerical conversion factor, enabling one to express a given physical quantity in terms of various units. No deep conceptual role is any more attributed to the constant, since the synthesis it symbolizes is, so to speak, achieved from the start. In other words, the theoretical status of a universal constant decreases as its practical importance increases. *A* good example of this situation is afforded by the classical thermodynamical constants J and k. The first one served to unify heat with work through the relationship $W = JQ$, while the second one

[§] Just ask Wikipedia for "rapidity" and "particles" and find many examples. See for instance C.-Y. Wong, *Introduction to High-Energy Heavy-Ion Collisions* (World Scientific 1994), p. 24

showed that temperature was but a statistical aspect of kinetic energy, as expressed by $E = kT$ (up to some numerical factor depending on the number of degrees of freedom). Of course, as emphasized earlier, J and k not only introduced new concepts, but whole new theories: thermodynamics for the first one, statistical mechanics for the second. We are so accustomed today to these ideas that they are incorporated into the implicit background of physical theory. Theoreticians almost automatically choose a convenient system of units such that $k = 1$, since they know that energy and temperature, or work and heat in fact *are* (now) but a single concept. In such a way, these constants grad ually fade out of sight in quite a literal sense: less and less are they written in formal expressions. From this point of view, it is seen that J and k indeed are universal constants, in the very same fundamental way as h or c. Only does our long collective practice of the concepts they express enable us to forget about their nature and to consider them as mere conversion factors. It must be said that such a process today is well under way concerning h and c. While all textbooks and articles in the first decades of the 20th century kept a detailed record of all h's and c's in their formulae, it is the common use today to take them as unity, which only means adopting a more adapted system of units. This convention has become almost tacit in the recent years, so that, except perhaps at the educational level, it will soon be obvious, that there is no difference of nature between h or c, on the one hand, and k or J, on the other.

This then is the ordinary fate of universal constants: to see their nature as concept synthesizers being progressively incorporated into the implicit common background of physical ideas, then to play a role of mere unit conversion factors and often to be finally forgotten altogether by a suitable redefinition of physical units. Once this is realized, one may well ask how many of these forgotten universal constants are lying around. Let us recall here the "Parable of the Surveyors" due to Taylor and Wheeler [21]:

«Once upon a time, there was a Daytime surveyor who measured off the king's lands. He took his directions of north and east from a magnetic compass needle. Eastward directions from the centre of the town square he measured in meters (x in meters). Northward directions were sacred and were measured in a different unit, in miles (y in miles). His records were complete and accurate and were of often consulted by the Daytimers.

Nighttimers used the services of another surveyor. His north and east directions were based on the North Star. He too measured distances eastward from the centre of the town square in meters (x' in meters) and sacred distances north in miles (y' in miles). His records were complete and accurate. Every corner of a plot appeared in his book with its two co-ordinates, x' and y'.

One fall a student of surveying turned up with novel openmindedness. Contrary to all previous tradition, he attended both of the rival schools operated by the two leaders of surveying. At the day school, he learned from one expert his method of recording the location of the gates of the town and the corners of plots of land. At night school, he learned the other method. As the days and nights passed, the student puzzled more and more in an attempt to find some harmonious relationship between the rival ways of recording location . He carefully compared the records of the two surveyors on the locations of the town gates relative to the center of the town square.

In defiance of tradition, the student took the daring and heretical step to convert northward measurements previously expressed always in miles, into meters by multiplication with a constant conversion factor, K. He then discovered that the quantity $[(x_A)^2 + (Ky_A)^2]^{1/2}$ based on Daytime measurements of the position of gate A had exactly the same numerical value as the quantity $[(x'_A)^2 + (Ky'_A)^2]^{1/2}$ computed from the readings of the Nighttime surveyor for gate A. He tried the same comparison on the readings computed from the recorded positions of gate B, and found agreement here too. The student's excitement grew as he checked his scheme of comparison for all the other town gates and found everywhere agreement. He decided to give his discovery a name. He called the quantity $[(x)^2 + (Ky)^2]^{1/2}$ the *distance* of the point (x, y) from the centre of town. He said that he had discovered the *principle of the invariance of distance;* that one gets exactly the same distances from the Daytime co-ordinates as from the Nighttime co-ordinates, despite the fact that the two sets of surveyors' numbers are quite different.»

Now we may realize that there is at least a most important universal constant in the simplest of all physical theories, namely plane Euclidean geometry. This constant expresses the theoretical assertion of space isotropy, enabling us to synthesize the concepts of northward and eastward distances into the single general concept of distance, independent of the orientation. Obviously, the numerical value of the constant K is without physical significance and due to mere historical contingencies. Its very existence, however, is a fundamental aspect of geometry. It is quite clear that, in due time, both distances, eastward and northward, in the town of the parable came to be measured with the same unit, so that the constant K disappeared and the discovery of the bright student faded into oblivion: was it not an "obvious" result?

This parable was intended by its authors to stress, and rightly so, the analogous role played today by the constant c with respect to space-time. But the parable of the surveyors also compels us, conversely, to unearth many of these forgotten universal constants, incorporated as they are into what now seems to be immediate truth, but was once the object of a lengthy and difficult working out. Another purely geometrical example is given by the evaluation of areas. Indeed let us choose a unit of area, as the area of some arbitrary plane figure, for instance an average human hand palm. It is then a “law of physics” (or a theorem of geometry) that the area A of a square with a side of length L, measured with some length unit, for instance the foot, is given by $A = \alpha L^2$, where α is some universal constant (with our units its value is roughly $\alpha = 0.11$ hand palm per squared foot). One *then* redefines the unit of area as the area of the square with unit side, so that α vanishes from sight. It should not be forgotten, however, that this constant expressed the now “obvious” synthesis of areas with squares of lengths. The same could be said of volumes, of course, and is not without factual relevance today. After all, the Anglo-Saxon traditional units of volumes, gallons or pints, are *not* defined by cubing the lengths units, foot or inch; the universal constant β entering the relationship $V = \beta L^3$ has not been taken as unity. We will see below that even within the scientific metric system, the constant β cannot be forgotten altogether. Now it should be clear that many such hidden universal constants lie at the core of the main statements of classical physics, at most in the oldest theories such as geometry or Newtonian mechanics, in which a long practice has led to the complete incorporation of their significance at an all-implicit level. The absence of universal constants in this part of physics is but an apparent privilege of old age. One might thus classify the universal constants (type-C above) into three subclasses according to their historical status:

i) the *modern* ones, such as h and c, the conceptual role of which is still dominant,

ii) the *classical* ones, such as k or J, which today appear essentially as unit conversion factors, their conceptual role having become almost implicit,

iii) the *archaic* ones, which have been so well assimilated and digested as to become totally invisible.

2·3. *The point of view of practice.* The story, however, is not that simple. It is only from the theorist's point of view that the life of a universal constant reaches the happy end of such a drift into the Nirvana of unity and oblivion.

The experimentalists working in the laboratory, when making measurements, must use concrete definitions of their units and cannot at will identify two operationally independent standards as the theorists on the paper do. It is a fact that, whatever fundamental system of units is adopted, based on the theoretical knowledge of the time, the use of units belonging to various other previous systems adapted to such and such domain of physics cannot be eliminated together. There are two reasons for this state of affairs. The first one is historical social inertia, which, for instance, forces the experimental physicists on the West side of the Atlantic to plan and order the nuts, bolts, plates, rods, etc. of their apparatus, by stating their dimensions in feet and inches rather than in metres and centimetres. The universal constant x entering the relationship $L_{us} = L_{EU}$ between the length of some object in the United States and the length of the same one in Europe (so that the subscript "EU" refers to us, while "US" refers to you) thus can be taken as unity in principle — but in principle *only.* This constant, once more, does expresses a fundamental law of physics, namely the homogeneity of space, enabling one to define the concept of length of an object independently of its location. But there is a second reason for the persisting of nonorthodox units, which is due to the nature of experimental physics proper, of which metrology is a fundamental aspect. Once a system of units is chosen (such as the modern International System of Units), every unit of a physical magnitude not belonging to the fundamental ones may be derived from the fundamental units. However, these derived units are often defined in such a way as to be of a very awkward use, or even as to lack the required precision, in a given domain of physics. Easier and better measurements may be done with the use of an independently defined local system of units. It then remains as a task for experimental metrology to relate these local units to the fundamental system through the experimental determination of conversion factors, which, as shown before, are nothing but genuine universal constants. A simple example may be given here. Before 1964, the litre as a unit of volume was defined independently of the length unit (metre) as the volume of a kilogram of water at 4°C. The relationship between volumetric measurements in litres and linear measurements in metres thus required the experimental determination of the universal constant in the relationship $V = \beta L^3$ between volumes and lengths; the value of this constant was β

= 1.000028 ± 0.000004 litre·dm^{-3}, which, of course, could be taken as unity — by the theorist[**]. It must be pointed out that the "noble" universal constant *c* is not, different in principle from the "trivial" β above. At the experimental level considered here, it is to be realized that the possibility of direct measurements of *c* only comes from it being also a type-A constant ; a direct determination of the velocity of photons thus leads to the value of *c*. But if there did not exist particles with zero mass (or approximately zero [9]), such a measurement would be impossible and *c* would have to be indirectly determined by relating measurements of electrostatic and magnetostatic quantities, or of lengths and frequencies. Such indirect determinations of *c* in the past sometimes have yielded the better values available at the time [22]. The present point in fact has been expressed by the best craftsmen themselves, such as Cohen and DuMond [23].

Exactly as the practical imperatives of experimental physics prevent a naive dismissing of "classical" constants by a change in unit conventions, more general social conditions can impose the persistence of "un- natural" archaic constants. Two simple historical examples of such a situation can be offered.

The first one is, once more, the question of volume measurements. Its scientific, metrological aspect, discussed earlier, corresponds to a much more general fact, valid since the highest antiquity; namely volumes usually are *not* determined by geometrical means from length measurements. Indeed, most volume determinations in practical life concern flowing materials, liquid or granular, such as beverages or grains (solids are mainly considered according to their weight). This is why independent volume units, defined by the capacity of some standard containers, have been the rule until the advent of the metric system (and thereafter — see above). For instance, in the Anglo-Saxon system, it has been said, pints are not related to inches. As a consequence, even though, as far as order of magnitudes are concerned, the volume unit usually has been comparable to the cube of the length unit, there was no real need to redefine these units in such a way that the appropriate universal constant be unity[††]. Surface measurements offer a comparable instance, with special units

[**] Nowadays, the litre has been redefined so as to be identical with the cubic decimeter (see http://www.bipm.org/jsp/fr/ViewCGPMResolution.jsp?CGPM=12&RES=6).

[††] An account of this situation has been given by Casimir in a

for land areas, although it is much less marked than for volumes since most determinations of surfaces have been done through length measurements by geometrical means, in all known historical periods.

The second example is furnished by the constant expressing the homogeneity of space, that is, the possibility of using the same units of length at each and every point of space. Apart from the case of the Anglo-Saxon system, this constant is now almost universally taken as unity, due to the international adoption of the metric system. This is a recent occurrence on the historical scale, however, and beyond a doubt came about very much later than the theoretical understanding of that possibility. The point is that such a unification was not necessary since space indeed was heterogeneous, socially, if not geometrically. Localized and rather autonomous social entities, from tribes to cities, were the rule in the human society until less than a few centuries ago. The progressive unification of the social space is related to the rise of merchant and industrial capitalism. It is the need of this new social order which brought about the redefinition of local units of lengths, so that the corresponding universal constant took on the status of an archaic one[‡‡].

3. – The case of the vanishing constants.

3·1. *How to vary a constant.* Universal constants not only play a role as standards of definition and measurement for physical quantities. They are further used as standards of validity for physical theories. This aspect is usually summarized by statements such as: "Galilean relativity is obtained from Einsteinian relativity in the limit $c \to \infty$", or "quantum mechanics goes into classical mechanics when Planck constant vanishes". Now these clearly are rather loose assertions, which are of formal significance at most, as they bear upon purely mathematical limiting processes imposed to the equations of the

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humorous parable illustrating the problem concerning electrical units inside dielectric media [24].

[‡‡] Let us not forget, though, that there are some domains where, for good reasons, horizontal and vertical distances are not measured with the same unit, such as air travel, where height are expressed in feet and lengths in miles. Better not to confuse them! More generally, quite a number of recent technological accidents have been due to errors in units conversions. Thus, in September 1999, the NASA Mars Climate Orbiter craft was lost due to a ground based computer software which produced output in non-SI units of pound-seconds (lbf s) instead of the metric units of newton-seconds (N s) used by the craft [25].

theory. But in the real world, the universal constants take on definite values and one is not free to change them at will. A better way of expressing these ideas is to assert the validity of Galilean relativity (respectively, classical mechanics), whenever c (respectively, h) can be considered as very large (respectively, small). One, however, has to be more accurate: large (or small) with respect to what? In the case of relativity, it is usually stated that the Galilean theory holds good, whenever the velocities are small with respect to c. But this is a necessary condition only, and it may be shown that it is not sufficient. Lorentz transformations with velocities small compared to c are approximated by Galilean ones, only for spatio-temporal intervals (or, more generally, four-vectors) which are of "large timelike" type, that is, such that $\Delta x << c\Delta t$. In the opposite case, that is for intervals (or four-vectors) of of "large spacelike" type, that is, such that $\Delta x >> c\Delta t$, an alternative limiting behaviour is obtained, giving rise to a second "nonrelativistic" limit of the Poincare group, the Carroll group [26] (see below). True, the space-time intervals concerned by such transformations, as, for instance, the interval between your reading of the next comma here and now and the emission within a second of a photon from some far-away star at a distance of two thousand light-years from home [27], are between events with no possible causal connection, precisely because of the large spacelike nature of these intervals. The Carroll group thus, necessarily applies to an acausal world (hence its name) and its physical relevance is dubious, to say the least (unless tachyons exist, the "nonrelativistic" properties of which could then be described through a Carrollian theory).

However, the very existence of the Carroll group, a well-defined and consistent mathematical, if not, physical object, serves to point out the necessity of a more stringent statement about the condition of validity for the Galilean approximation in relativity theory. It is to be required in effect that *all* relevant physical quantities with the dimensions of velocity ($\mathcal{L}\mathcal{T}^{-1}$) be small compared to c, that is, not only actual velocities of moving objects, but ratios of spatial to temporal intervals, of energies to momenta, etc. The necessity of such a general explicit condition may have been blurred by the narrow interpretation of c as a mere velocity (see above). Once it is recognized as a truly universal constant, it clearly acts as a standard of comparison for all physical quantities with the same dimensionality. At least is this requirement imperative if one is to set up a consistent theory. Weaker requirements may be sufficient to deal with specific situations. As an example, consider the transformation properties of electromagnetic fields under Lorentz transformations. If the velocity is low enough compared to c, the following formulae hold good:

$$\boldsymbol{E}' = \boldsymbol{E} - \boldsymbol{v}\times\boldsymbol{B}\ ,\ \ \boldsymbol{B}' = \boldsymbol{B} - c^{-2}\boldsymbol{v}\times\boldsymbol{E}. \qquad (1)$$

Now these formulae cannot fit into a full theory of electromagnetism in agreement with Galilean relativity [28]. In such a theory, two types of electromagnetic fields may exist with respective transformation properties:

$$\boldsymbol{E}' = \boldsymbol{E} - \boldsymbol{v}\times\boldsymbol{B},\ \boldsymbol{B}' = \boldsymbol{B} \qquad (2)$$

or

$$\boldsymbol{E}' = \boldsymbol{E},\ \boldsymbol{B}' = \boldsymbol{B} - c^{-2}\boldsymbol{v}\times\boldsymbol{E}. \qquad (3)$$

It is clear that (2) or (3) is valid depending on the fact that, in addition to $v << c$, one has either $E/B >> c$ or $E/B >> c$. These remarks, obviously, are related to the idea stressed earlier that a universal constant does not underlie a single concept, but a whole theoretical framework.

The situation has been clearer in that respect for quantum mechanics. Since h was never confused with a type-A constant (the "spin of light"…), from its universal nature it was rightly inferred that it had to be small compared with all relevant physical quantities with the dimension of an "action" (dimensionality $\mathcal{M}\mathcal{L}^2\mathcal{T}^{-1}$) for classical mechanics to be approximately valid. There is, however, a number of unsolved problems about the relationship of quantum theory to its classical limit(s), as will be mentioned below.

Another way of expressing the smallness (or largeness) of some universal constant in a given physical situation is to consider the units appropriate to the description of that situation, that is, if they exist at all, units such that all of the physical quantities take on "reasonable" values, spanning a limited range around unity. If the universal constant, when expressed with these units, is very small (or very large), then the approximate theory is valid which corresponds to the limit in which the constant goes to zero (or to infinity). This is clearly the case in the two examples mentioned up to now, where h takes on a very small value and c a very large one, when expressed in any system of units adapted to our daily experience (whether it be SI, or CGS, or the traditional Anglo-Saxon non-metric system).

The last remark, trivial as it may seem when applied to our modern familiar and revered universal constants *h* and *c*, may be of some help in understanding the historical reasons for the emergence, and later subsidence, of most universal constants, including the classical and archaic ones. Indeed, for *c* to appear as a universal constant, it was necessary for experimental investigation to come to grip with some phenomenon where at least one combination of physical quantities with dimension $\mathcal{L}\mathcal{T}^{-1}$ was comparable to *c*. This required a stage in the development of experimental techniques which was not reached until the 17th century with the first. measurements of the velocity of light [22]. Spatio-temporal ratios were for quite a time the only magnitudes with the required dimensionality to be measured with the necessary precision, so that *c* could not appear but as a type-A constant: the velocity of light and nothing more. It was not well until the 19th century that other physical magnitudes, namely electromagnetic ones, could be measured with a sufficient precision. Magnetism, after electricity, was subjected to accurate definitions and measurements, and the remark was left to Kirchhoff and Riemann that the combination of electric and magnetic constants which in modern formulation we would write as $(\varepsilon_0\mu_0)^{-1}$ was quite close to the speed of light[§§]. This first hint that *c* could well be at least a type-B constant, characteristic of electrodynamics in general, was confirmed by Maxwell's achievement of a consistent theory. By the beginning of this century, experimental progress had been such as to yield a vast number of combinations with the dimension $\mathcal{L}\mathcal{T}^{-1}$, the values of which were no longer small with respect to *c*; not only ratios of space to time intervals, but also of electric to magnetic field strengths (starting with Hertz experiments on electromagnetic radiation), products of frequencies and wavelengths, ratios (square root of) of energies to masses (in the energetics of nuclear reactions), of energies to momenta (dynamics of charged particles), etc. As for *h*, its existence could not be inferred before the possibility of investigating phenomena where characteristic "actions" were small enough and could be determined with a sufficient precision. The first of these turned out in spectroscopic studies when the black-body spectrum was studied at temperatures such that the maximum in

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[§§] G. Kirchhoff, "Ueber die Bewegung der Elektricität in Leitern", *Ann. Phys. Chem.* **102**, 529-544 (1857).
G.F.B. Riemann, "Ein Beitrag zur Elektrodynamik", *Annalen der Physik und Chemie*, pg 131 (1867).

the energy of the emitted radiation fell into an accessible range of wavelengths; namely, for Wien's laws to be discovered, the combination $kT\nu^{-1} = kT\lambda c^{-1}$ of the physical parameters under control had to be small enough to become comparable with *h*. Then the photo-electric effect disclosed as well the presence of Planck constant when the emission of ultraviolet radiation was mastered: the ratio of the kinetic energy of liberated electrons to the frequency of the radiation, that is $E\nu^{-1}$, could be measured with an accuracy enabling it to be compared with *h* [12]. As far as classical universal constants are concerned, now it is seen why *J* could not appear before the 19th century. The theoretical definition and experimental measurement of heat had to be pushed up to the point at which heat quantities ΔQ could be exchanged *and* measured, such that $J\Delta Q$ would no longer be negligible compared to the amounts of work ΔW commonly occurring [29]. The development of heat engines, such as the steam machine first, the progresses of physiology as well and, simultaneously, the improvement of thermometry and calorimetry, then led to the recognition of the "equivalence" between heat and work — one should better say their synthesis into a broader concept of energy — by Joule, Mayer, Helmholtz and others [30].

The hidden character of what was called above the archaic universal constants is readily understood from the present point of view. From the very physiological characteristics and social practice of humanity it follows that isotropy of space, for instance, has probably been incorporated right from the beginning in the use of one and the same unit of length for measuring distances in all directions. By a blending of Taylor and Wheeler's earlier "Parable of the Surveyors" [21] with Abbott's well-known novel, *Flatland* [31], one could imagine, however, a science-fiction story of an almost-flat species of intelligent beings. From their direct experience of the world, they could apprehend it on a scale of , say, metres in horizontal planes, but micrometres only in vertical directions. They would certainly use respective units with such a 10^6 ratio for their daily needs in vertical and horizontal-length measurements. The equivalence of vertical displacements Δz and horizontal ones Δl through a very small universal constant $\eta = 10^{-6}\ \mu\text{m}^{-1}$ would only be discovered by building telescopes giving access to vertical distances much larger (in the ratio $10^6 = \eta^{-1}$) than the ordinary heights in this world;

alter- natively, it could be brought to light by the investigation of microscopic horizontal displacements. Conversely, it is clear that two-dimensional physical theories, whether they be purely conceptual exercises or approximate descriptions of some physical phenomena, are in the same relationship with full three-dimensional theories, as, for instance, Galilean relativity to Einsteinian one, or classical physics to quantum physics. For such a two-dimensional theory, say, to be valid, all quantities with the dimensions of the ratio between a "vertical" and a "horizontal" length must be small compared with the universal constant η. In our conventional systems of units, of course, η is equal to unity and we recover a more customary statement for the validity of such approximate theories.

3·2. *The limitations of limits.* Let us now come back to our conventional modem universal constants in order to examine more closely the significance of limits such as $h \to 0$, or $c \to \infty$. We have already stressed that the constants in fact are constant and that, physically, the limits obtain when the dimensional ratios of the relevant physical quantities, say A/B, are small (or large) compared with the universal constant K which relates (synthetises) the two quantities A and B. It remains true that, in the formal expressions of the theory, this corresponds to considering the dimensionless ratio KB/A as large (or small), a situation which may be obtained as well giving a large (or small) value to the "constant" K. But several comments must be made to emphasize the limitations of these limit processes, which must be handled and interpreted with some care, if one is to avoid, or to correct, misunderstandings and delusions; the unicity, singularity and validity of such limiting processes will be investigated in turn.

i) *Unicity.* Contrarily to a naive idea, a given theory does not necessarily possess a unique more restricted theory as a limit. This is most clearly seen when the theory is expressed in its natural units, in which the universal constant is taken as unity to fully express the concepts of which it underlies the synthetic nature. For then, there is no longer any apparent dimensional constant that might go to zero, or infinity. One has to deal directly with the relevant ratios of physical quantities (dimensionless here), the choice of which must be guided by physical considerations. In other words, there are several inequivalent ways to reintroduce a constant in the theory, and several corresponding theories when the constant is eliminated through a limit process. The different possibilities deal with differing physical situations. A simple example of this point is furnished by Einsteinian relativity, as already mentioned. Let us write the Lorentz-transformation formulae for space-time intervals (Δx, Δt) in units such that $c = 1$

$$\Delta x' = \gamma\,(\Delta x - v\Delta t) \qquad (4a)$$

$$\Delta t' = \gamma\,(\Delta t - v\Delta x) \qquad (4b)$$

where, as usual, we have defined $\gamma = (1\text{-}v^2)^{-1/2}$, v being the velocity of the transformation. It is immediately apparent that the condition $v << 1$ is *not* sufficient to yield the Galilean transformations

$$\Delta x' = \Delta x - v\Delta t \qquad (5a)$$

$$\Delta t' = \Delta t. \qquad (5b)$$

One must require in addition that $\Delta x << \Delta t$, that is that the intervals be of large timelike type. This second condition is necessary for the second term in the equation (4b) to be neglected. If it does not hold, one may only write

$$\Delta x' = \Delta x - v\Delta t \qquad (6a)$$

$$\Delta t' = \Delta t - v\Delta x \qquad (6b)$$

These, as already said, may be useful approximate formulae in some circumstances, but cannot be the basis of a consistent relativity theory, since they obey no group law and hence no principle of relativity. Now the obvious symmetry of (4a) and (4b) of (4a) suggests a second limit, when $v << 1$ and $\Delta x << \Delta t$. These low-velocity transformations of large spacelike intervals read

$$\Delta x' = \Delta x \qquad (7a)$$

$$\Delta t' = \Delta t - v\Delta x. \qquad (7b)$$

They do obey a group law, defining the so-called Carroll group, already alluded to [26]. The Einsteinian relativity thus possesses two quite different "non-relativistic" limits, the Galilean and the Carrollian ones. That the second one cannot derive from the usual limit $c \to \infty$ is due to the fact that the conventional dimensional expression of the Lorentz transformation is such as to exclude a Carrollian situation right from the beginning. Indeed the replacements $v \to v/c$ and $\Delta t \to c\Delta t$ which enable one to recover the usual expression from (4) are such that $\Delta x / \Delta t \to \Delta x / c\Delta t$ which necessarily goes to zero (large timelike type) along with $v \to v/c$, when $c \to \infty$, leading to the Galilean transformations. If one was to "dimensionalise" the Lorentz transformations (4) through the replacements $v \to v/c'$, but $\Delta x \to c'\Delta x$, now the limit $c' \to \infty$, would yield the Carrollian transformations (7). The point clearly is that the conventional limit $c \to \infty$ is of a rather tautological nature, since it corresponds to following the evolutionary track of relativity theory in a time-reversed order. It is no surprise that it brings one back to the point of departure, that is the Galilean theory. If one is to study the possible limits of a theory, one must start from this theory as such, expressed within its autonomous system of concepts and intrinsic units. Once more it is seen how much the universal constants, even in the very most technical formulae, bear the mark of the historical developments of physics. The Carroll group probably is of little physical interest, so that the above considerations might seem of academic significance. A study of the nonrelativistic (Galilean) approximations to Maxwell's equations, however, leads in much the same technical way to realise that there exists two relevant physical limits [28]. There are two Galilean electromagnetisms, depending on whether it is the ratio of electric to magnetic fields, E/B, that is supposed to be small (in dimensionless units), or its inverse. Not surprisingly, one of these limits deals essentially with electric effects, the other one with magnetic effects. It must be mentioned, however, that both go beyond conventional electrostatics and magnetostatics in that they include, for instance, induction phenomena. It is to be said also that possible, more complicated and more interesting Galilean theories of electromagnetism exist which are *not* limits of Maxwell theory, in very much the same way that there exist "nonrelativistic" relativity groups which are not limits of the Lorentz group, such as the Newton groups [32]. A very elementary example of the existence of two different limits for a given theory is given by ordinary three-dimensional geometry. In writing the spatial interval $(\Delta r)^2 = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2$, a universal constant expressing the commensurability of horizontal and vertical lengths may be re-introduced in either of two ways. One may rescale the vertical lengths according to the replacement $\Delta z \to H\Delta z$, or the horizontal ones by $\Delta x \to H'\Delta x$, $\Delta y \to H'\Delta y$. The limits $H \to 0$ and $H' \to 0$, respectively, give rise to a two-dimensional plane geometry (see our previous parable) or to a one-dimensional linear one. I will comment below on the case of quantum mechanics and the limit $h \to 0$, since the situation is much less clear. However, it is well known in advance that quantum theory at least has *two* classical limits, dealing respectively with waves and corpuscles.

ii) *Singularity.* Kuhn has argued that the history of science proceeds through "scientific revolutions, in between which scientific activity would consist of "normal science" [33]. These revolutions would bring about the replacement of old paradigms by new ones, such that the ideas and concepts would undergo radical changes. For instance, mechanics is supposed to be so affected by the Einsteinian revolution that our ideas on space-time kinematics and dynamics have nothing· in common any more with those of Newtonian physics. Such strong statements, obviously, are contrary to all our experience as inquiring and teaching physicists. The difficulty here is that of the apparent dilemma between a continuous view of the history of science which would deny any qualitative change, and a discontinuous one which, ultimately fails to interpret, the process of change from one stage to the other. This is not the place to attempt a global evaluation of Kuhn's sociological history of science. One restricted aspect of his views, however, is closely related to the present investigation, namely the nature of the relationship between two successive paradigms in a given scientific domain. Taking as an example Kuhn's one of Einstein *vs.* Newton mechanics let us try to put it into perspective. This historical perspective, it, must he emphasized first, needs a backwards look. Obviously, the relationship between some physical theory and a more general successor cannot be studied until the generalization has succeeded. It is then necessarily from the point of view of the new, more encompassing, paradigm that the old one is to be judged. There is no vantage point, outer to both, from which their borderline could be seen and the transition analysed. We have to assess Newtonian mechanics starting from the Einsteinian one. In other words, the epistemological approach is necessarily opposite to the chronological one. It may be suggested then that this approach is that of a *singular* limit, in the mathematical sense. This statement, first, is certainly true at the factual level. Indeed the restricted theories, Galilean relativity or, more generally, Newtonian mechanics are obtained from the modern more general "relativistic" theory by a limit process which is necessarily singular. If it were not, the change would amount to a simple rescaling, without any conceptual modification. It is only in the limit in which c goes to infinity, and not when it is arbitrarily large but finite, that the old theory is recovered. In the case of relativity theory, a definite mathematical framework exists, the theory of contraction of groups [34], showing how a continuous family of group isomorphisms depending upon some parameter may tend towards a singular limit, whereby a new, nonisomorphic, group is obtained. But the idea of the old paradigm as a singular limit of the new one is proposed here in a wider, metaphorical sense, as well. It may help understanding how the transition from one to the other, as expressed by the vanishing (or infinity) of some universal constants, brings about qualitative changes into the conceptual tools of the trade. Indeed, if a universal constant brings about the synthesis and the unification of two previously unconnected concepts, its vanishing must be shown to give rise to the converse dis- junction, clearly a very singular phenomenon. This is the only way to under- stand, for instance, how the quantum energy-pulsation branches off into classical particle energy and classical wave pulsation.

As another example, Einstein mechanics knows of only two conserved quantities, energy and momentum, while Newtonian mechanics imposes the further conservation of mass, but introduces another, nonconserved quantity, namely internal energy. In this case, clearly, it is the Einsteinian mass, which in the Galilean limit yields both a conserved mass m and nonconserved internal energy U; of course Einstein's role was precisely to operate the inverse synthesis through the relationship $U = mc^2$! Let it be clear, however, that the singularity may be that of a coalescence of concepts as well as of a disjunction, since we deal here with the converse processes to both the syntheses and the splittings described above. But this aspect is rather trivial here, consisting for example in the merging of the rapidity φ with the velocity $v = \tanh\varphi$ in the Galilean limit. A final remark, at a more restricted technical level, derives from the mathematical singularity of these limit processes, as considered through universal constants. Much care must be exercised in investigating the limit of some theoretical expression when a universal constant is washed out by letting it go to zero or to infinity. In particular, the units used to write this expression should not depend on the constant itself. Obvious as it may seem, this rule is violated, for instance, by the numerous statements to the effect that "the magnetic moment of the electron, namely $\mu = eh/2mc$, is due to a relativistic phenomenon, because it is seen to depend on c" But this expression for μ is valid only in a system of units where the units of electric and magnetic field strengths are identical, which, as emphasized earlier, cannot be consistent with a Galilean theory. With different units, such as the SI ones for instance, the magnetic moment reads $\mu = eh/2m$

and, being independent of *c*, should be indifferent to the divergences of opinion between Galilei and Einstein. Indeed, it may be shown that the correct value of the moment obtains as well in a minimal Galilean theory of quantum particles with spin ½ as in Dirac theory [35] (of course, this is because spin itself is still much less an Einsteinian concept). This result extends to higher values of the spin [36]. Simple dimensional considerations show, on the other hand, that higher multipole electromagnetic moments, such as exist for high-spin particles, do vanish in the Galilean limit in which $c \to \infty$ (provided one consider elementary, structureless, particles, and not, for instance, composite systems such as the deuteron, of which the quadrupole moment owes nothing to Einstein, of course). Similar considerations may help in understanding the nature of the spin-orbit coupling for atomic electrons. It is usually said that the Thomas precession factor, due to an Einsteinian effect, halves the conventional coupling of the spin to the apparent magnetic field generated in the electron frame by the Coulomb field of the nucleus. It is very difficult, however, to accept that such a definite factor of ½ might go to 1 in the limit $c \to \infty$! A more rigorous analysis, in fact, shows that both terms are due to Einsteinian relativity (or else, that in a more complex Galilean theory, they could both exist but be numerically independent). In other words, before assessing the nature of a given effect or a property as due to the specificity of some theory because of its disappearance in the limit theory obtained when the relevant universal constant vanishes, a careful dimensional analysis of the problem is necessary, which requires explicitly disentangling some commonly used conventions of units, when they precisely rely on the theory the limits of which one is to test.

Let us end this section by an unfortunately half-baked idea which might be doomed to failure as an actual program, but should at least serve to underlie the highly singular nature of the limit processes on universal constants. The point is that, as already mentioned, we usually consider these limits in full knowledge of the resulting theory we want to obtain. The surprise, then, is meagre. Some exceptions have been mentioned (Carroll group, Galilean electromagnetism). But consider a more complicated process in which two universal constants simultaneously are pushed out of the theory. Specifically, let the dimensionless "fine structure" constant of electrodynamics $\alpha = e^2/hc$ keep its value, while $c \to \infty$ and $h \to 0$ simultaneously. The result, if there is one, should be a Galilean classical electrodynamics in which only all the quantities of Einsteinian quantum electrodynamics depending on α would keep their values, such as the ratio of the electromagnetic moment to its bare value and all other results of the renormalization program. True, this bare value, that is $\mu = eh/2m$, goes to zero with h, but why not to try computing directly the ratio of the dressed to the bare value, or even rescale m as well? It is very clear that such a theory, challenging as it is, requires a most careful analysis and re-writing of quantum electrodynamics; its singularity is certainly such as to brave any brute-force investigation.

iii) *Validity.* The last point to emphasize is that the existence of a well-defined formal limit for a theory when some universal constant vanishes is in no ways sufficient to guarantee the physical relevance of this limiting theory. The Carrollian kinematics [26] here offers an elementary illustration, since it has probably no applicability whatsoever in physical situations. But quantum mechanics offers a much richer and deeper example [14]. Indeed it is a surprise to realize, after almost a century of quantum theory, how little is known on its classical limits. Even at the formal level, things are far from being clear. It is empirically and historically known that quantum theory has resulted from the transcending synthesis of classical wave theory and particle mechanics. One should then be able to recover both these theories as limits of quantum theory now taken as such. The classical particle mechanics limit has received some attention, and various illustrations, from the Ehrenfest theorem to the JWKB approximation, or the relationship with Hamiltonian formalism, may be given of the transition. Things are much less clear on the other side, concerning classical wave theory. Indeed, since the vanishing of the *same* universal constant h seems implied in both limits, some additional assumption has to be made. From the empirical point of view, it is to be realized that an approximate classical particle behaviour may be exhibited by all quantum particles under specific circumstances: bubble or spark chambers thus exhibits "trajectories" and "collisions" for electrons as well as for photons and stranger particles yet. In contradistinction, approximate classical wave behaviour is shown but by boson assemblies, the one important example here being that of the electromagnetic field. It becomes clear then that the classical wave limit requires considering an indefinitely increasing number of particles while Planck constant vanishes. This point has being given too little attention [37].

Finally, it is to be emphasized that the existence of such formal limits is by no means a guarantee of the applicability of the approximate theory, or theories, thus obtained. Much more detailed assumptions have to be made if one is to understand and control at the theoretical level the approximate validity of a given limiting theory, even though it may be tested empirically. Concerning the case of quantum mechanics for instance, we know today that macroscopicity is *not* a sufficient condition for classicality, as is demonstrated by the existence of macroscopic quantum effects (in superfluids, for instance). It is not a surprise then that the very existence of ordinary, hard and stable matter, as approximately described by the classical mechanics of solid bodies, requires a very deep analysis at the quantum level in order to be understood from first principles [38]. It may serve as a useful conclusion then by reminding us that understanding the role of the physical constants is but the beginning of a concrete physical analysis, and only helps in asking the *right* questions — which are now *left* to be answered.

[1] See the interesting, although hardly compatible with the views presented here, paper by M. J. Duff, L. B. Okun and G. Veneziano, *J. High Energy. Phys.* 03, 023 (2002) and arXiv:physics/0110060v3.

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