THE MEANINGS OF PLANCK'S CONSTANT

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Abstract. This paper purposes to discuss the many roles played by Planck's constant in our understanding of quantum theory. It starts by retracing the rise of h to the status of a universal constant, which was certainly not obvious to every one at the beginning. The rather advanced views of Planck himself in that respect are stressed, particularly concerning the meaning of h as a 'quantum of action'. It is shown that this point of view, applied to statistical considerations, leads to a very interesting expression of the Pauli principle (Heisenberg-Pauli inequalities). The orthodox Copenhagen interpretation, which sees h as a 'gauge of indeterminism', is then challenged and reformulated in terms of a standard of quanticity. But the deep meaning of Planck's constant (in keeping with that of any fundamental constant), is to underly the synthesis leading to the new and specific concepts of quantum theory. It is then pointed out that the change from Planck's constant h to Dirac's $\hbar$, is much more than a convenient shortcut, as it enables one to obtain correct quantum order-of-magnitude estimates by heuristic considerations. Finally, it is stressed that the role of Planck's constant is by no means limited to the microscopic domain, and examples are given of its manifestations in the macroscopic (human scale) and megascopic (astronomic scale) domains.

Contents

1 Introduction

The *value* of h is $6,\ldots\times10^{-34}$ in the SI unit system — or 2π in the trade system ($\hbar = 1$).
The *history* of h starts with Planck's (quantum) jump in 1900, comes into full light in 1905 with Einstein, and takes momentum in 1924 with de Broglie.
The *role* of h is crucial in many areas of physics, from particle physics to solid state.
But what is to be said of its *meaning*, or rather, as I wish to stress the plural, of its *meanings* ?

Questions of meaning are often dismissed in a rather patronizing way by working physicists as "mere philosophy", as opposed to serious research, that is, theoretical formalism and experimental results. Yet, the purpose of science is (or should be) not only to predict or verify numbers and facts, but to assess and elaborate ideas, without which there is no real "understanding". It is crucial in that respect to recognize that almost any theoretical notion of physics, and certainly each important one, is prone to various interpretations and is endowed, in the course of time, with widely different meanings, or even with conflicting ones. This can be seen most clearly in the terminological diversity associated with some of these notions, and the ensuing confusion. A worse situation still is that of ancient ways of speaking, kept just out of mental laziness, while their initial strict meaning has long been forgotten. Think for instance of the so-called 'displacement current' in Maxwell equations (which was indeed for Maxwell — at the beginning — a true electric current in the ethereal substance, while we now think of it as a genuine field term), or consider the so-called 'velocity of light' (the fundamental role of which as a scale standard for the structure of space-time would amply justify a less specific name — for instance, Einstein constant). The importance of these remarks lies in the obvious but overlooked fact that neglecting the diversity of meanings and failing to discuss it explicitly may have strong adverse effects, not only on the spreading of knowledge (in teaching — see the example of the displacement current, or popularizing science — see the example of the velocity of light), but on its development as well; one does not know in advance which of the competing views, if any, will show the greatest fecundity.
I now come to the case of Planck's constant, for which I will discuss, without any claim to completenesss, a handful of the very different meanings it has been given. The argument will be set up in the framework I have proposed for understanding and classifying physical constants in general[1]. One has first to distinguish between fundamental constants, which are to be taken as basic elements of our knowledge, and derived, or phenomenological ones, which we know to be explainable (in priciple at least) from the fundamental ones; the electron charge or Newton's constant clearly belong to the first category, while Rydberg's constant or the proton mass (related to the quark masses and coupling constants) belong to the second one. The fundamental constants in turn may be classified under three headings: 1) specific properties of particular objects (say, the electron mass), 2) characteristics of whole classes of phenomena (say, the elementary electric charge which measures the strength of electromagnetic interactions), 3) universal constants which enter universal theories, ruling *all* physical phenomena (say, the limit velocity). I should stress that these distinctions should by no means be taken as giving a closed and atemporal classification; quite the contrary, they permit a detailed discussion of the historical changes in the role and meaning of the constants, as will now be seen.

2 The rise of h to universality

Planck's constant nowadays is clearly taken as a fundamental constant, and even a universal one, since quantum theory is thought to be universally valid; accordingly, there is no doubt that h in principle enters the treatment of any physical phenomenon. In many cases, a classical approximation leading us to forget its underlying presence is possible, but that is

[1] Jean-Marc Lévy-Leblond, 'On the Conceptual Nature of the Physical Constants', *Riv. Nuovo Cimento* 7 (1977), 187.

another question (and a difficult one at that, with some surprises in store as will be seen at the end of this paper).

Now, this universality was far from characterizing h at the beginning of its life.

The constant h first appeared in Planck's paper on the blackbody spectrum. It is perhaps not without interest to ask first why Planck chose to denote his new constant by this very letter; such questions, concerning the choices of symbols in the formalism of physics, certainly deserve more attention than they usually receive, as the answers could shed some light on the intellectual (and sometimes psychological) processes at the core of theoretical innovation. In the present case, it must be remarked that in its first printed occurrence (in october 1900), Planck's formula is written in terms of the wavelength :

$$e_\lambda = C\lambda^{-5}(e^{c'/\lambda T} - 1)^{-1}\ , \tag{1}$$

with constants C and c' (taken from the previously known Wien's formula) having a purely empirical meaning. It will take a few weeks for Planck to discover a theoretical explanation of this formula[2]. He considers the entropy of a 'resonator', starting from the statistical formula

$$S = k \ln \mathcal{N} \tag{2}$$

where the now so-called 'Boltzmann constant' appears in physics *for the first time*, and proceed to compute S under the hypothesis that the exchange of energy between radiation and matter is quantized with an 'Energieelement' given by:

$$\varepsilon = h\nu\ , \tag{3}$$

leading at last to the formula

$$u_\nu = 8\pi h c^{-3} \nu^3 (e^{h\nu/kT} - 1)^{-1}. \tag{4}$$

It is to be stressed that Planck introduced the *two* constants h and k. Later on, he would repeatedly complain that 'Boltzmann's constant' was in fact the *other* Planck's constant. In any case, the simultaneous appearance of h and k sheds some light, although a rather trivial one, upon the process of their denomination; it looks as if Planck (who seems never to have given any explanation whatsoever on this point) just chose the two first (and related) letters which did not yet bear too heavy a symbolic role in the accepted conventions of physics.

What is more important for the theme of our discussion, is that Planck, commenting upon the expression (3) of the 'energy element' and the general formula (2) for the entropy, explicitly stated that

"Hierbei sind h und k universelle Constante."

Contrarily to the usual view of Planck as rather old-minded and reluctant towards the modern aspects of quantum theory (an assessment which could be argued to reason considering his positions in the following decades), this statement about the universality of h and k certainly proves a more advanced stand than that of most of his contemporaries, for whom the fundamental nature of h was far from obvious, not to speak of its universality.

Indeed, it must be remembered that for quite a few years, h appeared only in considerations related to radiation theory, from the blackbody spectrum (Planck, 1899-1900) to the photoelectric effect (Einstein, 1905). It was only natural, then, to think of h as specifically ruling electromagnetic phenomena. In fact, not until Einstein's 1907 paper on the specific heat of solids, giving a first example of quantum statistical theory, did extend effectively its realm beyond radiation theory. It is all the more interesting to note, first, that Einstein himself, in his 1905 paper[3], uses neither the expression 'Planck's constant' (which was used for the first time rather late, probably by Millikan around 1915), nor even a specific symbol! Although he of course refers to this original paper by Planck, he does not write the 'Plancksche formel' (4), but expresses it in the more empirical form:

$$\rho_\nu = \alpha\nu^3 (e^{\beta\nu/T} - 1)^{-1} \tag{5}$$

where neither h nor k appear. It is even more striking to look at Einstein's formula for the photoelectric effect, since he writes:

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2 Max Planck, 'Ueber das Gesetz der Energieverteilung in Normaspektrum', *Ann. d. Phys.* 4 (1901), 553.

3 Albert Einstein, 'Über einen die Erzeugung und Verwandlung des Lichtes betreffenden heuristichen Gesichtspunkt', *Ann. d. Phys.* 17 (1905), 132.

"Die kinetische Energie solcher Elektronen ist $(R/N)\beta\nu - P$."

(where P is the extraction potential). This hiding away of h can certainly be taken as displaying at least some skepticism concerning the relevance of the theoretical derivation by Planck, as well as about the fundamental role of the constants h and k.

The idea was rather common in these days that h was not a fundamental constant, expressing the inception of a radically new theory, and that it could be explained away by some mechanical model leading to a more or less classical explanation of the quantization of energye exchanges. Born later on recalled the 'apple-tree model of Planck's quantization formula', an admittedly farcical model, which was discussed at the time[4]. Imagine an apple-tree with the property that the stems of the apples decrease with their height above ground; more specifically, let us suppose that the length l of the stems is inversely proprtional to the square of their height H, that is, $l \propto H^{-2}$. Then, the frequency of free oscillations of the apples considered as pendula is $\nu = (2\pi)^{-1}(g/l)^{1/2} \propto H$. Suppose now that the tree is shaking in the wind. Pressure waves with frequency ν will excite specifically the oscillations of those apples which lie at the corresponding height $H \propto \nu$. These apples only will fall to the ground, transferring to it an energy $E = mgH \propto \nu$; in other words, the exchange of energy between the tree and the ground is quantized in apple-units with an energy $E = \hbar\nu$, where the constant $\hbar$ may be expressed in terms of more fundamental quantities (the apple mass, the acceleration of gravity, and the constant of the length-height relationship).

In a more serious vein, there were several quite explicit attempts to relate h with suposedly more fundamental magnitudes of the atomic realm, like the Haas model based on the Thomson atom (which was favorably quoted by Sommerfeld as late as 1911). Einstein himself made a start in the same direction in 1909. He remarked that h had the same physical dimensionality as the combination e^2/c and looked upon this coincidence as indicating the possibility of a specifically electromagnetic mechanism[5] (note that this happened the very same year when he himself showed, through his theory of the specific heat of solids, h to have a general relevance, beyond purely electromagnetic phenomena!). Lorentz, in a letter dated 6 May 1909, expressed serious doubts, based upon the numerical gap (three orders of magnitudes) between the two expressions, writing "I could imagine to have a factor of 4π or so intervening, but a factor 900, that is really too much." To what Einstein carefreely answered that "a factor like $6(4\pi)^2$ is not so extraordinary."

But unless someone produces a theory of the fine structure constant α yielding its numerical value, thus giving h as equal to $2\pi\alpha(e^2/c)$, but above all explaining its seemingly ubiquitous role (beyond electromagnetism), we have better think of Planck's constant, in the very terms of Planck himself, as a universal constant. However, within this general conception, there remains a variety of possible views, as will now be seen.

3 The quantum of action

A first concrete understanding of h as a universal constant was put forward by Planck himself. He had for long noticed that h had the dimensionality of classical 'action'. In his contribution to the 1911 Solvay conference, he introduced the very fecund idea that h in fact defined the magnitude of irreducible quantities of action, "elementaren Wirkungsquanten" in Planck's original terms. There was born the wording 'quantum of action' for h. This new vision of quantization was in turn expressed as the existence of elementary areas in phase space, that is, cells A with finite extension:

$$\iint_A dp\,dq \cong h\,. \tag{6}$$

Planck was clearly aware that this point of view called for a renunciation to all attempts at classical interpretations of h (of the type mentioned above). He wrote, in his Solvay paper: "The framework of classical dynamics, even if combined with the Lorentz-Einstein principle

4 See Max Jammer, *The Conceptual Development of Quantum Mechanics* (McGraw Hill, 1966).

5 Albert Einstein, 'Zum gegenwärtigen Stand der Strahlungsproblem', *Phys. Zeits.* 10 (1909), 185.

of relativity, is too narrow to account for all those physical phenomena which are not directly accessible to our coarse senses."[6]

Planck's idea was seized upon by Sommerfeld, who, inspired by what he called the "most fortunate" naming of h as a quantum of action, related it more precisely to Hamilton's action function which enabled him to develop the Old Quantum Theory.

Later on, the formal development of quantum theory was to be built upon another, more general, meaning of h (see below). Nevertheless, despite the limitations of the 'quantum of action' point of view, it has not lost its fecundity, and can be put to good use, at least as a heuristic tool for understanding some aspects of quantum behviour. As an example, it leads to a very picturesque and useful way of expressing the Pauli principle, as I now proceed to show.

The Heisenberg-Pauli inequalities

Consider a system of N one-dimensional particles, classical ones to start with. The state of the system can be represented at a given time, by a collection of N points in the two-dimensional one-particle phase-space *(p,q)*. Contemplate now a system of N quantons (quantum 'particles'). In accordance with Planck's idea about the quantum of action, an individual state can no longer be represented by a point, but is to be associated to an extended cell with an area of order h, or rather $\hbar$; indeed, as will stressed below, one should definitely use the Dirac constant $\hbar := h/2\pi$ as soon as numerical evaluations are contemplated. Denoting by Δp and Δq respectively the extension of an individual cell in momentum and position respectively (in order of magnitude), the expression (6) for the minimal area of the cell constitures but a rewriting (and an interpretation) of the Heisenberg inequality:

$$\Delta p \Delta q \succ \hbar \qquad (7)$$

(where the symbol $\succ$ denotes inequality up to a constant numerical factor of order unity). The collective state then consists of an ensemble of N such cells. Let us suppose now that the quantons are identical fermions. The effect of Fermi statistics may be expressed by the Pauli exclusion principle which requires that no two individual states be identical. It is traightforward to translate this constraint as requiring the N cells now to be disjoint (Figure 2c). One sees vividly how the Pauli principle requires the collective state of the system to have a much greater minimal extension in phase-space. Indeed, for a system with N quantons *not* obeying the Pauli principle, the inequality (7) for the individual regions in phase-space applies as well to the total region, since nothing prevents the individual states to be one and the same. For identical fermions however, the total region, consisting of non-overlapping cells, must have a minimal area at least equal to N times that of an individual cell, leading to an inequality characterizing the collective extensions in position and momentum of a system with N identical fermions in one dimension:

$$\Delta p \Delta q \succ N\hbar . \qquad \text{(1-D)} \qquad (8)$$

I call it the 'Heisenberg-Pauli inequality'. It can easily be generalized to three (or more…) dimensions. The individual cells now have a minimal volume $\hbar^3$ and the collective state with N identical fermions must occupy a region of volume at least $N\hbar^3$. Hence the Heisenberg-Pauli inequality in three dimensions:

$$\Delta p \Delta q \succ N^{1/3}\hbar . \qquad \text{(3-D)} \qquad (9)$$

It should be emphasized that this heuristic derivation may be given a fully rigorous form, and that the inequality (9) may be proved formally[7]. The best way to understand the Heisenberg-Pauli inequalities (8) and (9), is to compare them to the standard Heisenberg inequality (7), and to consider that the effect of the Pauli principle is to replace the fundamental quantum constant $\hbar$ by an effective fermionic one, $\hbar_f^{(d)}(N) = N^{1/d}\hbar$, where d is the dimensionality of space (which shows, by the way, that the effect of the exclusion principle is all the more powerful, the lower is the dimensionality of space).

[6] Max Jammer, *op. cit.*

[7] Jean-Marc Lévy-Leblond, 'Generalized Uncertainty Relations for Many-Fermion Systems', *Physics Letters* 26A, 540 (1968).

Let us put to use the Heisenberg-Pauli inequality in a simple example, namely, the evaluation of the ground-state energy of an N electron atom. We start with the hamiltonian

$$H = \sum_{k=1}^{N} \frac{p_k^2}{2m} - \sum_{k=1}^{N} \frac{Ze^2}{r_k} + \sum_{k=1}^{N} \sum_{l=1}^{k-1} \frac{e^2}{|r_k - r_l|} \ , \tag{10}$$

where the notations are obvious enough. Denote by $\tilde{p}$ and $\tilde{r}$ respectively the average, order-of-magnitude wise, of the momentum and radial distance of the electrons in a stationary sate of the atom. The energy of the state may then be written as:

$$E \approx Z \frac{\tilde{p}^2}{2m} - Z^2 \frac{e^2}{\tilde{r}} \ , \tag{11}$$

the terms in the right-hand side being taken up to a numerical constant [the potential term results from the combined coulombic attraction of the electrons by the nucleus and the mutual repulsion of the electrons, both being, within a constant, of order $Z^2 e^2 / \tilde{r}$, but the first one dominating, as shown by the very existence of atoms, hence the negative sign in (11)]. If quantum theory is now invoked but no supplementary assumption is made, its effects may be simulated by the Heisenberg inequality (7), leading to:

$$E \succ Z \frac{\tilde{p}^2}{2m} - Z^2 \frac{E^2}{\hbar} \tilde{p} \ . \tag{12}$$

Minimizing this expressions with respect to $\tilde{p}$, we obtain an evaluation of the ground-sate energy:

$$E_0 \overset{?}{\approx} - Z^3 \frac{me^4}{\hbar^2} \ . \tag{13}$$

This is *wrong*, as shown for instance by the fact that it predicts an average size for the atoms (corresponding to the value of the electronic radial distance in the ground state (13))

$$\tilde{r}_0 \overset{?}{\approx} Z^{-1} \hbar^2 / me^2 \ , \tag{14}$$

which would make the atoms shrink very fast with the number of the electrons, contrariwise to evidence. The reason of the discrepancy clearly is our neglect of the fermionic nature of the electrons. We now simply account for the Pauli principle by replacing Planck's constant in the result (13) by its effective fermionic value $\hbar_f^{(3)} = Z^{1/3}\hbar$, which lead to the correct estimate:

$$E_0 \approx -Z^{7/3} \frac{me^4}{\hbar^2} \ . \tag{15}$$

The rather unexpected exponent 7/3 usually is obtained from elaborated calculations, for instance in the Thomas-Fermi approximation. The present derivation is both simpler and more transparent.

The same reasoning may be applied to give a heuristic discussion of the all important question of the saturation of Coulombic forces in macroscopic matter (namely, the constancy of the binding energy per particle with respects to the size of the system)[8], shedding some light on the very sophisticated discussion by Dyson, Lenard, Lieb, and Thirring. This type of argument also finds applications in the discussion of gravitationally bound systems, explaining the transition from rocks (dominated by Coulomb cohesive forces) to planets (dominated by Newton cohesive forces), and even to white dwarfs[9,10]. We will return later on to the macroscopic manifestations of Planck's constant.

4 Gauge of indeterminism or standard of quanticity?

[8] Jean-Marc Lévy-Leblond & Françoise Balibar, *Quantics (Rudiments)* (North-Holland,1990), chapter 7.

[9] Jean-Marc Lévy-Leblond, 'Mécanique quantique des forces de gravitation et stabilité de la matière', *J. de Phys.* 30 (1969), C3-43.

[10] Jean-Marc Lévy-Leblond, 'Quantum Theory at Large', in E. Beltrametti & JMLL eds, *Adv. in Quantum Phenomena*, (Plenum, 1996), pp. 281-295.

With the advent of the new quantum mechanics, and in the wake of its discussions by the Copenhagen school, a novel meaning was attributed to Planck's constant. At the 1927 Solvay conference, Bohr and Heisenberg presented a paper on matrix mechanics and the probability interpretation, in which they delivered the following pronouncement:

> "The real meaning of Planck's constant h is this: it constitutes a universal gauge of the indeterminism inherent in the laws of nature owing to the wave-particle duality." [11]

I have no time here to delve into a detailed criticism of the notions of indeterminism and duality. Let us only note that they carry a rather pessimistic view of quantum mechanics, giving an essentially negative role to Planck's constant. Indeed, it is still rather common, in discussions of the Heisenberg inequalities, formulated in terms of alleged 'uncertainties', to view h as a quantitative measure of the limits imposed to our knowledge of nature. This assessment does not fit well, to say the least, with the positive use of the Heisenberg inequalities in heuristic approaches (as exemplified above), which, far from leading us to stumble against some alleged intrinsic limitations of our knowledge, enable us to gain some intuitive feeling of quantum phenomena[12]. More generally, the subtle, specific and constructive roles of h in many areas of modern quantum physics, should relegate the emphasis on a supposed quantum indeterminism to the historical record. For it is only from a classical point of view that the 'wave-particle duality' and its associated 'indeterminism' make sense. Once we fully accept quantum ideas, the whole terminology loses its meaning.

A modern formulation would rather consider h as a 'standard of quanticity', a universal standard against which to assess the necessity of putting quantum theory to work, a signpost of the limits of validity of classical approximations. The criterion is the following : when considering some phenomenon, evaluate the relevant quantities with the dimension of an action ($\mathcal{M}\mathcal{L}^2\mathcal{T}^{-1}$) and compare them with Planck's constant ; if they are much larger than h, then a classical theory will yield a good approximation, if not, the recourse to quantum theory is compulsory. Note that this criterion by no means is equivalent to setting up a distinction between microscopic and macroscopic quantities. This is clearly emphasized by Feynman[13], when he discusses how the so-called Heisenberg principle (not a principle, in fact, since it derives from the basic formalism of quantization) 'protects' quantum theory by virtue of the universality of h. Indeed, he shows that a contradiction would appear if one were to apply the Heisenberg inequalities only in the microscopic world (say, to the electron in a two-slit experiment) and not in the macroscopic one (say, to the screen in the same experiment). And we now know many macroscopic quantum effects, which make the point obvious (see also below).

5 The concept synthetizer

Traditionally, the Planck-Einstein relationship

$$E = h\nu \qquad (16)$$

has been interpreted in the spirit of the so-called 'wave-particle duality', as associating an energy (mechanical quantity linked to the particle aspect) with a frequency (vibratory quantity linked to the wave aspect). From a modern point of view, it rather appears that this relationship in fact expresses the emergence of a new and original concept, which trancends both the classical concepts of energy and frequency. This is the role generally played by universal contants, which may be best characterized as *concept synthetizers* [14]. Consider for instance the Joule constant J appearing in the formula $W = JQ$ expressing the mechanical equivalent of heat. Its role goes far beyond that of a simple unit conversion coefficient to which it is often unduly relegated; indeed, this very formula upholds the general concept of energy, surpassing the previously unrelated notions of work and heat. The same could be said of the Einstein constant c^2, which, through the formula $E = c^2m$ (purposely written here in an unfamiliar guise), gives rise to a new concept of 'relativistic' energy, beyond the previously

[11] See Max Jammer, *The Philosophical Development of Quantum Mechanics* (Wiley, 1994), p. 114.
[12] Ref. 8, chapter 3
[13] *The Feynman Lectures in Physics*, vol. 3, chapter 2, (Addison-Wesley).
[14] See ref. 1.

separate notions of mass and energy. From that point of view, the relationship (16) is not to be interpreted as linking two classical concepts, but rather as transcending them through their synthesis, and establishing a new concept with a broader scope. In fact, a new name could profitably have been given to this new concept, stressing its originality. That the *quantum* energy thus constructed differs from the classical energy is sufficiently shown, for instance, by the fact that a quantum system is not, in general, characterized by a single and well-defined value of its energy, but rather by a whole numerical spectrum. In other words, energy and frequency, through the relationship (16) appear as but two particular facets of a more general notion, each of which being the only visible one from either one of two specific viewpoints (the wave and the particle aspects, respectively). The present interpretation reverses the traditional one, as it stresses, instead of a duality of *classical* objects (wave vs particle), the unity of quantum objects — which would deserve a specific name, 'quantons' (as suggested by M. Bunge) probably being the best proposal[15].

A universal constant in general does not synthetizes a mere pair of notions, but unifies whole theoretical structures, and brings about several syntheses. This is certainly true for Planck's constant, since, besides bringing together energy and frequency as time-related notions, it also ties the two space-related notions of momentum and wave-number according to the de Broglie relationship:

$$p = hk \qquad (17)$$

By the way, from this modern point of view, one may (naively) wonder why it has taken twenty years to go from the formula (16) to its homolog (17). A further step may be taken, remembering the existence of yet another conserved quantity in classical physics, besides energy and momentum, namely angular momentum. In the same way as, through the constant h, energy is linked to time frequency and momentum to space frequency, it is most natural to conjecture the relationship

$$L = h\mu \qquad (18)$$

where L is some component of the angular momentum, and μ the angular frequency of a rotationally periodic (or rather harmonic) phenomenon. That is to say, $\mu = 1/\alpha$ if α is the angular period. Now comes the bonus. Indeed, contrarily to space and time, which are represented by the real line, angles define a compact set, the circle. This means that an angular period cannot be arbitrary, but has to be a submultiple of the full-turn (daisies have an integral number of petals), i.e. $\alpha = 2\pi/m$ where m is some integer. Finally:

$$L = \hbar m, \qquad m \text{ integer} \qquad (19).$$

This line of reasoning may be extended to allow for half-integer values as well[16]. That the quantization of the angular momentum may be obtained from such a simple and deep argument clearly shows the interest of recognizing the nature of h as the universal quantum synthetizer.

The present point of view gives a solid and clear basis to the now customary choice of units in quantum theory, whereby Planck's constant, or rather the Dirac's constant $\hbar$ (see below), is taken as unity. Far from being a mere convenience, this choice, exhibitng the identification of energy and frequency, entails the acknowledgement that there is but one single quantal notion of energy-frequency. It is only from a macroscopic and limited point of view that our dealing with objects approximately described as particles or waves has led us to set up two separate notions, energy and frequency, which we now recognize as particular aspects of a more general concept.

6 From Planck's to Dirac's constant

It seems that the introduction of the constant $\hbar := h/2\pi$ is due to Dirac, in his book, *Principles of Quantum Mechanics*. By that time in effect, the dividing factor 2π had made its appearance in various expressions, from the rigorous form of the Heisenberg inequalities to

[15] Jean-Marc Lévy-Leblond, 'Quantum Words for a Quantum World', in D. Greenberger & al., *Epistemological and Experimental Perspectives on Quantum Physics* (Kluwer, 1999), pp. 75-87.

[16] Jean-Marc Lévy-Leblond, 'Quantum Heuristics of Angular Momentum', *Am. J. Phys.* 44 (1976), 719 , and ref. 8, chapter 2.

the quantization of angular momentum, so that the new notation brought a welcome simplification. What I want to stress here is that there is more to it than a handy convention.
A time periodic phenomenon is specified by its period T. However, this duration is rather too large if one is to define a characteristic time for the phenomenon, that is, the order of magnitude of a time interval during which the phenomenon changes 'notably', passing from small to large, for instance. For a harmonic phenomenon, the most natural choice corresponds to a change of phase of the order of 1 radian — which is indeed a characteristic angle, neither too small (a few degrees) nor too large (a full turn or so). The associated characteristic time then is $\tau = T/2\pi$, which leads to use the pulsation $\omega = 1/\tau = 2\pi/T = 2\pi\nu$ in theoretical expressions rather than the frequency $\nu = 1/T$ (the latter, on the other hand, being better adapted to experimental measurements, as it is more natural to count full cycles rather than radians). The same argument can be developed for a space periodic phenomenon, leading to the use of the 'reduced wave number' $\lambdabar = \lambda/2\pi$, as the characteristic length of the phenomenon, and the inverse quantity $\kappa = 1/\lambdabar = 2\pi/\lambda$ in the reciprocal space. This quantity unfortunately lacks an accepted appellation; it could well be named 'undulation', in perfect analogy with the time pulsation. The extension to angular periodicity, with period α, is immediate, leading to the 'angulation' $m = 2\pi/\alpha$ Using these characteristic quantities, the fundamental quantum relationship now read :

$$\begin{aligned} &E = \hbar\omega \quad (\text{time}) \\ &p = \hbar\kappa \quad (\text{space}) \\ &L = \hbar m \quad (\text{angle}) \end{aligned} \tag{20}$$

with $\hbar$ now appearing in the role of the quantum synthetizer.
The importance of these considerations lies in the fact that it is by using Dirac's $\hbar$, and not Planck's h, that reliable order of magnitudes estimates can be obtained by using dimensional analysis or other heuristic methods. For instance, dimensional analysis alone is sufficient to write the ionization energy of the hydrogen atom as $Ame^4/\hbar^2$ (with obvious notations) or its size as $A'\hbar^2/me^2$, where A and A' are dimensionless constants. In effect, these constants are of order unity, while they would be of order 40 ($= 4\pi^2$) if we had chosen h instead of $\hbar$ in these expressions. The basic principle of physics according to Wheeler, namely:

> "All dimensionless constants are of order unity, IF the characteristic magnitudes are correctly chosen (with allowance for exceptions)"

is indeed satisfied, but only if one is careful enough to systematically use Dirac's constant as the 'good' quantum constant.

7 The macroscopic level

The status of Planck's constant as the keystone of quantum theory by no means imply that its role is confined to the microscopic domain, and to atomic or subatomic phenomena. Despite the smallness of its numerical value in the SI unit system, many macroscopic phenomena, on the human scale, are governed by it. As major examples, consider:
— the blackbody radiation, indeed one of the problems at the origin of quantum theory ; Stefan's constant, which governs the amount of energy radiated by a heated body is (within a numerical constant) $\sigma \approx k^4/\hbar^3c^3$. Planck's constant thus rules the baker's or potter's owen, and, first of all, the amount of energy radiated by the Sun and received by the Earth, that is, the very possibility of life.
— the density of ordinary matter (a few grams per cubic centimeter) is in fact a microscopic quantity as well, since the average atomic volume is given by the size of Bohr's radius ; the density of matter in bulk, thanks to the saturation of Coulombic forces (a major quantum effect), is the same as that of individual atoms, to wit $\rho \approx M/(\hbar^2/me^2)^3$, where M is a representative atomic mass.

— the typical voltage of ordinary electrochemical batteries (a few volts[17]), is governed by the ionization energy of atoms, so that $V \approx q_e^{-1} me^4 / \hbar^2$; in other terms, one volt is worth one electron-volt per electron…

In fact, the role of Planck's constant extends beyond the macroscopic domain (human scale) well into the megascopic domain (astronomic scale). Indeed, cold enough stellar objects, such as white dwarfs or neutron stars, are governed by the quantum Fermi pressure of their constituents (electrons or neutrons respectively); it turns out that there is an upper bound to their size, which is known as the Chandrasekhar limit, corresponding to a critical number of nuclei of order $N_c \approx (\hbar c / GM^2)^{3/2}$ — which is, and not by chance, also the order of magnitude of the number of atoms ($\approx 10^{57}$) in a typical star such as the Sun. Less well-known is the fact [18] that the size of a life-bearing planet may be shown to be necessarily of order $R \approx (e^2 / GM^2)^{1/2} (\hbar^2 / me^2)$ and the maximal size of the living beings it hosts of order $l \approx (e^2 / GM^2)^{1/4} (\hbar^2 / me^2)$ — at least for ground animals.

It is appropriate to conclude with the following quotation by Maxwell (1870):

> "If we wish to obtain standards of length, time and mass which shall be absolutely permanent, we must seek them not in the dimensions or the motion or the mass of our planet, but in the wavelength, the period of vibration and the absolute mass of these unperishable and unalterable and perfectly similar molecules."

Maxwell was indeed right to call for a redefinition of our metrological sandards based on atomic (and therefore quantum) magnitudes. But we now know that the dimensions of our planet and our own are not arbitray, but closely linked to those of the 'molecules' — thanks to Planck's constant.

[17] It is particularly appropriate to mention this aspect here and now, as we celebrate in 2000 the bicentenary of the discovery of the battery by Alessandro Volta, who spent quite a time living and workin here in Pavia.

[18] Ref. 10.